Earthquakes happen all the time. Most earthquakes are never felt by anyone, but some can be natural disasters. They are devastating not only because they are powerful, but also because they are largely unpredictable.
Have you ever seen a funny picture or video online, and then it suddenly seems like everyone is talking about it? Social media and the internet have made it really easy to pass things along from person to person. You can pin, post, and share anything that you find interesting, thought-provoking, or funny with your friends all over the world. Your friends can in turn share it with their own friends, who share it with their friends, and before you know it, it seems like everyone in the world is exposed to it.

When something becomes extremely popular on the internet in a very short amount of time, it’s known as “going viral.” Trends can spread across the country or around the world in a matter of days. Some “viral” videos and pictures have produced overnight celebrities and inspired spin-offs in the form of books or TV shows.

What is your favorite “viral” video or picture that you’ve seen online?
Big Things Come to Those Who Wait!

Allison and Beth each receive $10 per week for doing chores for their neighbor. One day, Allison decides to try and increase her income using her knowledge of exponential growth. She proposes that her payment be changed to a penny, and then doubled each week thereafter.

1. Complete the table to represent the amount that Allison and Beth will earn each week.

<table>
<thead>
<tr>
<th>Week</th>
<th>Allison’s Income (dollars)</th>
<th>Beth’s Income (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01</td>
<td>10.00</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. How does Allison’s income change as the number of weeks increases?

3. Does Allison’s income represent an arithmetic or geometric sequence or series? Explain your reasoning and state the general formula.

4. Write an equation to represent Allison’s income after $n$ weeks.
5. What is the value of $a_n$ for $n = 0$? Does this value make sense in this problem situation?

6. If the pattern were to continue, how many weeks would it take for Allison to have a larger weekly income than Beth? Complete the table to show your answer.

<table>
<thead>
<tr>
<th>Week</th>
<th>Allison's Income (dollars)</th>
<th>Beth's Income (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

You can write the explicit formula for the geometric sequence $a_n = 0.01 \cdot 2^{(n-1)}$ in function notation using the properties of powers.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_n = 0.01 \cdot 2^{(n-1)}$</td>
<td>Explicit formula for a geometric sequence</td>
</tr>
<tr>
<td>$f(n) = 0.01 \cdot 2^{(n-1)}$</td>
<td>Rewrite in function notation</td>
</tr>
<tr>
<td>$f(n) = 0.01 \cdot 2^n \cdot 2^{-1}$</td>
<td>Product Rule</td>
</tr>
<tr>
<td>$f(n) = 0.01 \cdot 2^n \cdot \frac{1}{2}$</td>
<td>Definition of negative exponents</td>
</tr>
<tr>
<td>$f(n) = 0.01 \cdot \frac{1}{2} \cdot 2^n$</td>
<td>Commutative Property of Multiplication</td>
</tr>
<tr>
<td>$f(n) = 0.005 \cdot 2^n$</td>
<td>Associative Property of Multiplication</td>
</tr>
</tbody>
</table>

So, $a_n = 0.01 \cdot 2^{(n-1)}$ written in function notation is $f(n) = 0.005 \cdot 2^n$.

Recall that a geometric sequence, when written in function notation, is called an exponential function. The function gets its name from the variable in the exponent.
7. Calculate the income that Allison would earn per week in the:
   a. 15th week.
   b. 20th week.
   c. 24th week.

8. Predict the shape and characteristics of the graph that will model Allison’s income as a function of the number of weeks.
1. Beth is amazed at how quickly Allison was able to make a lot of money and decides that she wants in on the action. She asks her two friends, Quinton and Alisha, to help her come up with a plan.

**Quinton**
You could start with a dollar and ask for 50% more each week.

**Alisha**
You could start with a dollar and add another dollar each week.

Whose plan should Beth choose? Complete the table and graph to justify your reasoning. Round to the nearest hundredth.

<table>
<thead>
<tr>
<th>Week</th>
<th>Quinton's Plan (dollars)</th>
<th>Alisha's Plan (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Graph showing the amount earned over weeks for both plans.]
2. Write functions to represent Quinton’s plan, \( q(x) \), and Alisha’s plan, \( a(x) \).

3. Use your choice from Question 1 to determine how much Beth will earn in Week 10.

4. If Beth and Allison both start using their exponential model to earn income at the same time, who will earn a higher income in Week 12?

5. Use a graphing calculator to determine when Allison’s and Beth’s incomes will be equal. Does this make sense in this problem situation? Explain your reasoning.
6. Compare Allison’s and Beth’s function models.
   a. As the number of weeks continues to increase, whose model will earn them more per week?
   
   b. Consider the $a$- and $b$-values of the exponential functions if $y = ab^x$. How do they further support your claim?

**PROBLEM 3 Half-Life of Caffeine**

Simeon is studying for a big test and is trying to stay awake. He drank a 12-ounce can of Big Buzz Energy Drink that contains 80 milligrams of caffeine. He is wondering how long the caffeine will stay in his system if the caffeine has a half-life of 5 hours.

A **half-life** is the amount of time it takes a substance to decay to half of its original amount.

1. How much caffeine remains in Simeon’s system after 5 hours? After 10 hours? Explain your reasoning.

2. Complete the table to determine the amount of caffeine in Simeon’s system at each time interval.

<table>
<thead>
<tr>
<th>Time Elapsed (hours)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Caffeine in System (mg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Half-Life Cycles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. What is the initial amount of caffeine in Simeon’s system? What is the rate of decay?
4. Emily, Tyler, and Renee were asked to write an exponential function $A(t)$ to represent the amount of caffeine remaining in Simeon’s system after $t$ hours.

Emily

\[ A(t) = 80 \left( \frac{1}{2} \right)^{\frac{t}{5}} \]

The variable $t$ represents the number of hours, and the half-life occurs in 5 hour cycles, so I divided my exponent by 5.

Tyler

\[ A(t) = 80 \left( \frac{1}{2} \right)^{-\frac{t}{5}} \]

The variable $t$ represents the number of hours and since it’s a decay function, I made my exponent negative.

Renee

\[ A(t) = 80 \left( \frac{1}{2} \right)^{5t} \]

The variable $t$ represents the number of hours and I multiplied it by 5 to represent the half-life cycle of 5 hours.

a. Why is Tyler’s reasoning incorrect?

b. Why is Renee’s reasoning incorrect?

5. How much caffeine remains in Simeon’s system after 2 hours?

It may be helpful to substitute the values from the table to check each student’s function.
6. Kendra suggests that she can calculate the amount of caffeine remaining by rewriting the equation as $A(t) = 40^t$. Is Kendra correct? Explain your reasoning.

7. Use the table function of a graphing calculator to predict when the caffeine will be completely out of Simeon’s system. Does this make sense, given what you know about exponential functions? Explain your reasoning.

8. Approximately when will the amount of caffeine remaining in Simeon’s system be less than 1 milligram?

9. Use the properties of exponents to rewrite your function so that only the variable $t$ is in the exponent. What percentage of caffeine remains after each hour?
10. Suppose Simeon is taking an antibiotic that extends the half-life of caffeine to 8 hours. Write a function \( B(t) \) that models the amount of caffeine remaining under these new conditions.

11. Complete the table for the new half-life. Round to the nearest hundredth.

<table>
<thead>
<tr>
<th>Time Elapsed (hours)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Caffeine in System (mg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Half-Life Cycles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

12. How does the medication affect the amount of caffeine remaining in Simeon’s system?

13. Under these new conditions, approximately when will the amount of caffeine remaining in Simeon’s system be less than 1 milligram?

14. What generalization can you make about the effect of larger or smaller half-lives on substances?

Be prepared to share your solutions and methods.
Have you ever tried to remember a long list of things and ended up getting mixed up along the way?

There are lots of tried-and-true ways of memorizing things, and it all depends on what you’re trying to memorize. Some people like to make mnemonic devices, where the first letter in each word corresponds to something in the list they’re trying to memorize. You may have used one of these when you were learning the order of operations—Please Excuse My Dear Aunt Sally is a great way to help you remember parentheses, exponents, multiplication, division, addition, and subtraction. Some people try doing some sort of movement as they recite their list, so that they can use their muscle memory to help them. Some people like to use rhymes, some people use visualization, and some people rely on good old-fashioned repetition.

But there are some people who are just naturally skilled at remembering things. In fact, there are competitions held around the world to see who can memorize the most digits of pi. In 2005, Chao Lu of China set a world record by memorizing an incredible 67,890 digits of pi! It took him 24 hours and 4 minutes to accurately recite the digits, with no more than 15 seconds between each digit.

Do you have any memory tricks to help you remember things?
PROBLEM 1  I’ve Got the Power

1. Cut out the exponential graphs and equations and match them. Sort them into “growth” or “decay” functions, and tape them onto the graphic organizer in this lesson. Then, complete each table.

<table>
<thead>
<tr>
<th>Graph</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$f(x) = \left(\frac{1}{3}\right)^x$</td>
</tr>
<tr>
<td>B</td>
<td>$f(x) = 10^x$</td>
</tr>
<tr>
<td>C</td>
<td>$f(x) = 1.5^x$</td>
</tr>
<tr>
<td>D</td>
<td>$f(x) = 0.1^x$</td>
</tr>
<tr>
<td>E</td>
<td>$f(x) = 3^x$</td>
</tr>
<tr>
<td>F</td>
<td>$f(x) = \left(\frac{1}{3}\right)^x$</td>
</tr>
</tbody>
</table>

Think about how the base affects the graph of an exponential function!
<table>
<thead>
<tr>
<th>Growth</th>
<th>Decay</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( f(x) )</td>
</tr>
<tr>
<td>(-1)</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

### Growth

### Decay
2. Analyze the exponential growth and decay functions.
   a. What point do the graphs have in common? Why?

   b. Compare the equations of the six functions you just sorted. What differentiates an exponential growth from an exponential decay?

3. Sarah and Scott’s teacher asked them to each write a rule that would determine whether a function was exponential growth or decay, based on its equation.

   **Sarah**
   
   For exponential growth functions, $b$ is a value greater than 1, but for exponential decay functions, $b$ is a fraction or decimal between 0 and 1.

   **Scott**
   
   For exponential growth functions, $b$ is greater than 1. For exponential decay functions, $b$ is less than 1.

Why is Scott’s reasoning incorrect? Provide a counterexample that would disprove his claim and explain your reasoning.
4. What $b$-values for a function of the form $f(x) = b^x$ produce neither growth nor decay? Provide examples to support your answer.

5. Write an exponential function with the given characteristics.
   a. Increasing over $(-\infty, \infty)$
      Reference point (1, 6)
   
   b. Decreasing over $(-\infty, \infty)$
      Reference point (-1, 4)
   
   c. End behavior $\lim_{x \to -\infty} f(x) = 0$
      $\lim_{x \to \infty} f(x) = \infty$
      Reference point (2, 6.25)

6. Summarize the characteristics for the basic exponential growth and exponential decay functions.

<table>
<thead>
<tr>
<th></th>
<th>Basic Exponential Growth $f(x) = b^x, b &gt; 1$</th>
<th>Basic Exponential Decay $f(x) = b^x, 0 &lt; b &lt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asymptote</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercepts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>End Behavior</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intervals of Increase</td>
<td></td>
<td></td>
</tr>
<tr>
<td>or Decrease</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
PROBLEM 2  Let’s Compound Some Dough

Helen is opening her first savings account and is depositing $500. Suppose she decides on a bank that offers 6% annual interest to be calculated at the end of each year.

1. Write a function \( A(t) \) to model the amount of money in Helen’s savings account after \( t \) years.

2. Calculate the amount of money in Helen’s savings account at the end of 1 year?

3. How much money will be in Helen’s savings account at the end of 5 years?

4. Suppose that the bank decides to start compounding interest at the end of every 6 months. If they still want to offer 6% per year, how much interest would they offer per 6-month period?

Recall that the formula for compound interest is \( A = P(1 + r)^t \).
5. John, Betty Jo, and Lizzie were each asked to calculate the amount of money Helen would have in her savings at the end of the year if interest was compounded twice a year. Who’s correct? Explain your reasoning.

John
1st 6 months:
\[ A(t) = 500(1 + 0.03) \]
\[ A(t) = 515 \]
2nd 6 months:
\[ A(t) = 515(1 + 0.03) \]
\[ A(t) = 530.45 \]

Betty Jo
\[ A(t) = 500 \left( 1 + \frac{0.06}{2} \right)^2 \]
\[ A(t) = 530.45 \]

Lizzie
\[ A(t) = 2(500(1 + 0.03)) \]
\[ A(t) = 1030 \]

6. Write a function to model the amount of money in Helen’s savings account at the end of \( t \) years, compounded \( n \) times during the year.
7. Determine the amount of money in Helen's account at the end of 3 years if it is compounded:
   a. twice a year.
   b. monthly.
   c. daily.

8. What effect does the frequency of compounding have on the amount of money in her savings account?
PROBLEM 3  Easy “e”

Recall that in Problem 2, the variable $n$ represented the number of compounding periods per year. Let’s examine what happens as the interest becomes compounded more frequently.

1. Imagine that Helen finds a different bank that offers her 100% interest. Complete the table to calculate how much Helen would accrue in 1 year for each period of compounding if she starts with $1.

<table>
<thead>
<tr>
<th>Period of Compounding</th>
<th>$n =$</th>
<th>Formula</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yearly</td>
<td>1</td>
<td>$1 \left(1 + \frac{1}{1}\right)^{1\cdot1}$</td>
<td>2.00</td>
</tr>
<tr>
<td>Semi-Annually</td>
<td>2</td>
<td>$1 \left(1 + \frac{1}{2}\right)^{2\cdot1}$</td>
<td>2.25</td>
</tr>
<tr>
<td>Quarterly</td>
<td>4</td>
<td>$1 \left(1 + \frac{1}{4}\right)^{4\cdot1}$</td>
<td></td>
</tr>
<tr>
<td>Monthly</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weekly</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daily</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hourly</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Every Minute</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Every Second</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Make an observation about the frequency of compounding and the amount that Helen earns. What amount is it approaching?
The amount that Helen's earnings approach is actually an irrational number called $e$.

$$e = 2.718281828459045 \ldots$$

It is often referred to as the **natural base** $e$.

In geometry, you worked with $\pi$, an irrational number that was approximated as 3.14159265... and so on. Pi is an incredibly important part of many geometric formulas and occurs so frequently that, rather than write out “3.14159265...” each time, we use the symbol $\pi$.

Similarly, the symbol $e$ is used to represent the constant 2.718281... It is often used in models of population changes as well as radioactive decay of substances, and it is vital in physics and calculus.

The symbol for the natural base $e$ was first used by Swiss mathematician Leonhard Euler in 1727 as part of a research manuscript he wrote at age 21. In fact, he used it so much, $e$ became known as Euler's number.

The constant $e$ represents continuous growth and has many other mathematical properties that make it unique, which you will study further in calculus.

3. The following graphs are sketched on the coordinate plane shown.

   $f(x) = 2^x$, $g(x) = 3^x$, $h(x) = 10^x$, $j(x) = \left(\frac{3}{5}\right)^x$, $k(x) = 1.3^x$.

   a. Label each function.

   b. Consider the function $m(x) = e^x$. Use your knowledge of the approximate value of $e$ to sketch its graph. Explain your reasoning.
c. Using the functions $f(x) = 2^x$, $g(x) = 3^x$, $m(x) = e^x$, approximate the values of $f(2)$, $g(2)$, and $m(2)$ on the number line. Explain your reasoning.

PROBLEM 4 It Keeps Growing and Growing and Growing…

1. The formula for population growth is $N(t) = N_0e^{rt}$. Complete the table to identify the contextual meaning of each quantity.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Contextual Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_0$</td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td></td>
</tr>
<tr>
<td>$N(t)$</td>
<td></td>
</tr>
</tbody>
</table>

2. Why is $e$ used as the base?

3. How could this formula be used to represent a decline in population?
4. The population of the city of Fredericksburg, Virginia, was approximately 19,360 in 2000 and has been continuously growing at a rate of 2.9% each year.
   a. Use the formula for population growth to write a function to model this growth.
   b. Use your function model to predict the population of Fredericksburg in 2013.
   c. What value does your function model give for the population of Fredericksburg in the year 1980?

5. Use the table feature of a graphing calculator to estimate the number of years it would take Fredericksburg to grow to 40,000 people, assuming that the population trend continues.

Be prepared to share your solutions and methods.
Andy Warhol was an American pop artist whose work explored the relationship between artistic expression, celebrity culture, and advertisement. A recurring theme throughout Warhol’s art is the transformation of the mundane and commonplace into art. His most renowned images are silk-screened reproductions of Campbell’s soup cans and publicity photographs of pop culture icons like Marilyn Monroe and Elvis Presley.

Have you ever seen any of Andy Warhol’s work?
1. The two tables show four exponential functions and four exponential graphs.
   a. Match the exponential function to its corresponding graph, and write the function under the graph it represents.
   b. Explain the method(s) you used to match the functions with their graphs.

<table>
<thead>
<tr>
<th>Exponential Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = 10^x )</td>
</tr>
<tr>
<td>( h(x) = -10^x )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exponential Graphs</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Exponential Graph 1]</td>
</tr>
<tr>
<td>![Exponential Graph 3]</td>
</tr>
</tbody>
</table>
2. Analyze the graphs.
   a. Write an equation for \( h(x) \) in terms of \( f(x) \). Describe the transformation on \( f(x) \).

   b. Write an equation for \( g(x) \) in terms of \( f(x) \). Describe the transformation on \( f(x) \).

   c. Write an equation for \( j(x) \) in terms of \( f(x) \). Describe the transformation on \( f(x) \).

3. Determine the asymptotes, intervals of increase and decrease, and end behavior for each exponential function.

<table>
<thead>
<tr>
<th>Function</th>
<th>Asymptotes</th>
<th>Intervals of Increase and Decrease</th>
<th>End Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = 10^x )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( g(x) = 10^{-x} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( h(x) = -10^x )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( j(x) = -10^{-x} )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. How would the graph of \( k(x) = \left( \frac{1}{10} \right)^x \) compare to the graph of \( g(x) = 10^{-x} \)?
5. How do the transformations on \( f(x) \) affect the asymptotes, intervals of increase and decrease, and end behavior?

PROBLEM 2 Keep On Moving

Consider the functions \( y = f(x) \) and \( g(x) = Af(B(x - C)) + D \). Recall that the \( D \)-value translates \( f(x) \) vertically, the \( C \)-value translates \( f(x) \) horizontally, the \( A \)-value vertically stretches or compresses \( f(x) \), and the \( B \)-value horizontally stretches or compresses \( f(x) \).

Exponential functions are transformed in the same manner.

For an exponential function with a base of 2, the transformational function can be written as:

\[
t(x) = A \cdot 2^{B(x - C)} + D
\]

1. Given, \( f(x) = 2^x \), determine how \( A, B, C, \) or \( D \) transforms the graph of the graph of \( f(x) = 2^x \).
   a. \( g(x) = A \cdot 2^x \) for positive and negative values of \( A \).

   b. \( h(x) = 2^x + D \), for positive and negative values of \( D \).

   c. \( j(x) = 2^{x-C} \), for positive and negative values of \( C \).

   d. \( j(x) = 2^{Bx} \), for positive and negative values of \( B \).

2. Consider exponential functions with different bases, such as \( c(x) = 10^x \) and \( d(x) = e^x \). How will the values of \( A, B, C, \) or \( D \) transform the graphs of \( c(x) = 10^x \) and \( d(x) = e^x \)? Explain your reasoning.
3. Consider each transformation to \( f(x) = 2^x \). Describe the effects of each transformation, including effects on the key attributes of the graph. Then, sketch the transformed graph.

\[ g(x) = 5 \cdot 2^x \]

\[ p(x) = 2^{x-4} \]
c. \( k(x) = 2^x - 3 \)

\[ \begin{array}{|c|c|c|c|c|c|c|}
\hline
x & -8 & -6 & -4 & -2 & 0 & 2 \\
\hline
y & 8 & 6 & 4 & 2 & 0 & 2 \\
\hline
\end{array} \]

\[ \begin{array}{|c|c|c|c|c|c|c|}
\hline
x & -8 & -6 & -4 & -2 & 0 & 2 \\
\hline
y & 8 & 6 & 4 & 2 & 0 & 2 \\
\hline
\end{array} \]


d. \( q(x) = 2^x + 4 \)

\[ \begin{array}{|c|c|c|c|c|c|c|}
\hline
x & -8 & -6 & -4 & -2 & 0 & 2 \\
\hline
y & 8 & 6 & 4 & 2 & 0 & 2 \\
\hline
\end{array} \]

\[ \begin{array}{|c|c|c|c|c|c|c|}
\hline
x & -8 & -6 & -4 & -2 & 0 & 2 \\
\hline
y & 8 & 6 & 4 & 2 & 0 & 2 \\
\hline
\end{array} \]

e. \( j(x) = 2^x + 3 \)

\[ \begin{array}{|c|c|c|c|c|c|c|}
\hline
x & -8 & -6 & -4 & -2 & 0 & 2 \\
\hline
y & 8 & 6 & 4 & 2 & 0 & 2 \\
\hline
\end{array} \]

\[ \begin{array}{|c|c|c|c|c|c|c|}
\hline
x & -8 & -6 & -4 & -2 & 0 & 2 \\
\hline
y & 8 & 6 & 4 & 2 & 0 & 2 \\
\hline
\end{array} \]
f. \( h(x) = -5 \cdot 2^x \)

4. Consider the transformations to \( f(x) = 10^x \). Describe the effects of the multiple transformations, including effects on the key attributes of the graph. Then, sketch the transformed graph.

a. \( s(x) = -3 \cdot 10^{x+2} - 1 \)
b. $t(x) = 2 \cdot 10^{-x} + 3$

5. Consider the transformations to $f(x) = e^x$. Describe the effects of the multiple transformations, including effects on the key attributes of the graph. Then, sketch the transformed graph.

a. $w(x) = 4 \cdot e^{x+3} + 1$
b. \( z(x) = -8 \cdot e^{x^2} - 5 \)

The function \( f(x) = 3^x \) is shown. Recall the key characteristics of basic exponential functions, including a domain of all real numbers, a range of positive numbers, and a horizontal asymptote at \( y = 0 \).

6. Suppose that \( a(x) = f(x + 1) \).
   a. Describe the transformation on the graph of \( f(x) \) that produces \( a(x) \).
   
   b. Complete the table to determine the corresponding points on \( a(x) \), given reference points on \( f(x) \). Then, graph and label \( a(x) \).

<table>
<thead>
<tr>
<th>Reference Points on ( f(x) )</th>
<th>Corresponding Points on ( a(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1, \frac{1}{3}))</td>
<td></td>
</tr>
<tr>
<td>((0, 1))</td>
<td></td>
</tr>
<tr>
<td>((1, 3))</td>
<td></td>
</tr>
</tbody>
</table>
c. Determine the domain, range, and asymptotes of $a(x)$.

7. Suppose that $b(x) = f(x) + 1$.
   a. Describe the transformation on the graph of $f(x)$ that produces $b(x)$.

   b. Complete the table to determine the corresponding points on $b(x)$, given reference points on $f(x)$. Then, graph and label $b(x)$.

<table>
<thead>
<tr>
<th>Reference Points on $f(x)$</th>
<th>Corresponding Points on $b(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-1, \frac{1}{3})$</td>
<td></td>
</tr>
<tr>
<td>(0, 1)</td>
<td></td>
</tr>
<tr>
<td>(1, 3)</td>
<td></td>
</tr>
</tbody>
</table>

   c. Determine the domain, range, and asymptotes of $b(x)$. 
8. Suppose that \( c(x) = f(x) - 5 \).
   a. Describe the transformation on the graph of \( f(x) \) that produces \( c(x) \).

b. Complete the table to determine the corresponding points on \( c(x) \), given reference points on \( f(x) \). Then, graph and label \( c(x) \).

<table>
<thead>
<tr>
<th>Reference Points on ( f(x) )</th>
<th>Corresponding Points on ( c(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1, \frac{1}{3}))</td>
<td></td>
</tr>
<tr>
<td>((0, 1))</td>
<td></td>
</tr>
<tr>
<td>((1, 3))</td>
<td></td>
</tr>
</tbody>
</table>

![Graph of \( f(x) = 3^x \)]

C. Determine the domain, range, and asymptotes of \( c(x) \).
9. Suppose that \( d(x) = f(2x) \).
   a. Describe the transformation on the graph of \( f(x) \) that produces \( d(x) \).

   b. Complete the table to determine the corresponding points on \( d(x) \), given reference points on \( f(x) \). Then, graph and label \( d(x) \).

<table>
<thead>
<tr>
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<th>Corresponding Points on ( d(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1, \frac{1}{3}))</td>
<td></td>
</tr>
<tr>
<td>((0, 1))</td>
<td></td>
</tr>
<tr>
<td>((1, 3))</td>
<td></td>
</tr>
</tbody>
</table>

   c. Determine the domain, range, and asymptotes of \( d(x) \).

10. Analyze the transformations performed on \( f(x) \) in Questions 6 through 9.
   a. Which, if any, of these transformations affected the domain, range, and asymptotes?

   b. What generalizations can you make about the effects of transformations on the domain, range, and asymptotes of exponential functions?
11. Andres and Tomas each described the effects of transforming the graph of \( f(x) = 3^x \), such that \( p(x) = 3f(x) \).

Andres
\[
p(x) = 3f(x)
\]
The A-value is 3 so the graph is stretched vertically by a scale factor of 3.

Tomas
\[
p(x) = 3f(x)
\]
\[
p(x) = 3 \cdot 3^x
\]
\[
p(x) = 3^{x+1}
\]
\[
p(x) = f(x + 1)
\]
The C-value is -1 so the graph is horizontally translated 1 unit to the left.

a. Explain Andres’ and Thomas’ reasoning.

b. Determine the domain, range, and asymptotes of \( p(x) \).

c. What generalizations can you make about the effects of vertical dilations on the domain, range, and asymptotes of exponential functions?
1. Analyze the graphs of \( f(x) \) and \( g(x) \). Describe the transformations performed on \( f(x) \) to create \( g(x) \). Then, write an equation for \( g(x) \) in terms of \( f(x) \). For each set of points shown on \( f(x) \), the corresponding points are shown on \( g(x) \).

a. \( g(x) = \) ____________________

b. \( g(x) = \) ____________________
c. \( g(x) = \) _________________

2. The equation for an exponential function \( m(x) \) is given. The equation for the transformed function \( t(x) \) in terms of \( m(x) \) is also given. Describe the graphical transformation(s) on \( m(x) \) that produce(s) \( t(x) \). Then, write an exponential equation for \( t(x) \).

a. \( m(x) = 2^x \)
   \[ t(x) = 0.5m(x + 3) \]

b. \( m(x) = e^x \)
   \[ t(x) = -m(x) - 1 \]

c. \( m(x) = 6^x \)
   \[ t(x) = 2m(-x) \]

Be prepared to share your solutions and methods.