To use synthetic division to divide a polynomial $ax^2 + bx + c$ by a linear factor $x - r$, follow this pattern.

![Diagram showing synthetic division process]

You can use synthetic division in place of the standard long division algorithm to determine the quotient for $(2x^2 - 3x - 9) \div (x - 3)$.

<table>
<thead>
<tr>
<th>Long Division</th>
<th>Synthetic Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 3$</td>
<td></td>
</tr>
<tr>
<td>$x - 3$</td>
<td>3</td>
</tr>
<tr>
<td>$2x^2 + 3x - 9$</td>
<td>2 -3 -9</td>
</tr>
<tr>
<td>$2x^2 - 6x$</td>
<td>2</td>
</tr>
<tr>
<td>$3x - 9$</td>
<td>3</td>
</tr>
<tr>
<td>$3x - 9$</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$(2x^2 - 3x - 9) \div (x - 3) = 2x + 3$

1. Analyze the worked example.
   a. Write the dividend as the product of its factors.
   b. Why does the synthetic division algorithm work?

Notice when you use synthetic division, you are multiplying and adding, as opposed to multiplying and subtracting when you use long division.
2. Two examples of synthetic division are provided. Perform the steps outlined for each problem:

   i. Write the dividend.
   ii. Write the divisor. \( (x - r) \)
   iii. Write the quotient. \( \text{answer} \)
   iv. Write the dividend as the product of the divisor and the quotient plus the remainder.

\[
\begin{array}{r|cccc}
   & 1 & 0 & -4 & -3 \\
\hline
   2 & & 2 & 4 & 0 \\
   & 1 & 2 & 0 & -3 \\
\end{array}
\]

\[ r = 2 \]

\[ x^4 - 4x^2 - 3x + 6 = (x - 2)(x^3 + 2x^2 - 3) \]

\[
\begin{array}{r|cccc}
   & -3 & 2 & -4 & -4 \\
\hline
   2 & & -6 & 30 & -78 \\
   & 2 & -10 & 26 & -81 \\
\end{array}
\]

\[ x^4 - 4x^3 - 4x^2 - 3x + 6 = (x + 3)(2x^3 - 10x^2 + 26x - 81) + \frac{249}{x + 3} \]
3. Calculate each quotient using synthetic division. Then write the dividend as the product of the divisor and the quotient plus the remainder.

a. \( g(x) = x^3 + 1 \)
   \( r(x) = x + 1 \)
   
   \[
   \begin{array}{c|ccc}
   -1 & 1 & 0 & 0 \\
   -1 & 1 & 1 & -1 \\
   \hline
   1 & -1 & 1 & 0
   \end{array}
   \]

   \[= x^2 - x + 1\]

b. \( g(x) = x^3 + 8 \)
   \( r(x) = x + 2 \)
   
   \[
   \begin{array}{c|cccc}
   -2 & 1 & 0 & 0 & 8 \\
   -4 & 2 & 4 & -8 \\
   \hline
   1 & -2 & 4 & 0
   \end{array}
   \]

   \[= x^2 - 2x + 4\]

c. \( g(x) = x^3 + 27 \)
   \( r(x) = x + 3 \)
   
   \[
   \begin{array}{c|cccc}
   -3 & 1 & 0 & 0 & 27 \\
   -3 & 9 & -27 \\
   \hline
   1 & -3 & 9 & 0
   \end{array}
   \]

   \[= x^2 - 3x + 9\]

d. \( g(x) = x^3 + 64 \)
   \( r(x) = x + 4 \)
   
   \[
   \begin{array}{c|cccc}
   -4 & 1 & 0 & 0 & 64 \\
   -4 & 16 & -74 \\
   \hline
   1 & -4 & 16 & 0
   \end{array}
   \]

   \[= x^2 - 4x + 16\]

Do you see a pattern? Can you determine the quotient in part (d) without using synthetic division?
6. Analyze each division problem given \( f(x) = x^3 - 3x^2 - x + 3 \).

\[
g(x) = \frac{f(x)}{x - 1} \quad h(x) = \frac{f(x)}{2x - 2} \quad j(x) = \frac{f(x)}{3x - 3}
\]

a. Determine the quotient of each function.

\[
g(x) = \frac{x^3 - 3x^2 - x + 3}{x - 1}
\]

\[
\begin{array}{c|cccc}
 & 1 & -3 & -1 & 3 \\
\hline
1 & 1 & -2 & -3 \\
-2 & 0 & -3 & 0 \\
\end{array}
\]

\[
x = 1, \quad x^2 - 2x - 3 = x^2 - 2x - 3
\]

b. Use function notation to write \( h(x) \) and \( j(x) \) in terms of \( g(x) \).